

Gravitating BIon and BIon black hole with dilaton

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We construct static and spherically symmetric particle-like and black hole solutions with magnetic or electric charge in the Einstein-Born-Infeld-dilaton system, which is a generalization of the Einstein-Maxwell-dilaton (EMD) system and of the Einstein-Born-Infeld (EBI) system. They have remarkable properties which are not seen for the corresponding solutions in the EBI and the EMD system. In the electrically charged case, the electric field is neutralized at the origin by the effect of the diverging dilaton field. The extreme solution does not exist but we can take the zero horizon radius limit for any Born-Infeld (BI) parameter b . Although the solution in this limit corresponds to the ‘particle-like’ solution in the Einstein frame, it is not a relevant solution in the string frame. In the magnetically charged case, the extreme solution *does* exist for the critical BI parameter $\sqrt{b}Q_m = 1/2$. The critical BI parameter divides the solutions qualitatively. For $\sqrt{b}Q_m < 1/2$, there exists the particle-like solution for which the dilaton field is finite everywhere, while no particle-like solution exists and the solution in the $r_h \rightarrow 0$ limit becomes naked for $\sqrt{b}Q_m > 1/2$. Examining the internal structure of the black holes, we find that there is no inner horizon and that the global structure is the same as the Schwarzschild one in any charged case.

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The pioneering theory of the non-linear electromagnetic field was formulated by Born and Infeld (BI) in 1934 [1]. Their basic motivation was to solve the problem of the self-energy of the electron by imposing a maximum strength for the electromagnetic field. Although their attempt did not succeed in this regard, many kinds of solution, such as a vertex and a particle-like solution (BIon), which were constructed afterward in the model including BI term were of great interest. Some extensions of the BI type action to the non-Abelian gauge field are considered, although it is not determined uniquely because of the ambiguity in taking the trace of internal space [2]. One of the interesting recent results is that there exist classical glueball solutions which were prohibited in the

standard Yang-Mills theory [3].

Moreover, it was considered to include the self-gravitational effect of the isolated systems by putting the Einstein gravity [4]. Then self-gravitating particle-like solutions (EBIon) [6] and their black hole solutions (EBIon black hole) were discovered analytically under the static spherically symmetric ansatz [7,8]. The non-linearity of the electromagnetic field brings remarkable properties to avoid the black hole singularity problem which may contradict the strong version of the Penrose cosmic censorship conjecture in some cases. Actually a new non-linear electromagnetism was proposed, which produces a nonsingular exact black hole solution satisfying the weak energy condition [9,10], and has distinct properties from Bardeen black holes [11]. Unfortunately, the BI model is not this type.

Surprisingly it has been shown that the world volume action of a D-brane is described by a kind of non-linear BI action in the weak string coupling limit [12]. In particular, the bosonic D-3 brane will have a world volume action that is the BI action. In addition to this, BI action arises in the sting-generated corrections if one considers the coupling of an Abelian gauge field to the open bosonic string or the open superstring [13]. This system also includes the dilaton and the antisymmetric Kalb-Ramond tensor field, which we call the axion field. Hence it is a direct extension of the Einstein-Maxwell-dilaton-axion (EMDA) system, where the famous black hole solution was found with the vanishing axion (i.e., in the EMD system) by Gibbons and Maeda, and independently by Garfinkle, Horowitz and Strominger (GM-GHS) [14]. It is interesting that the non-linear electromagnetic field appears in string theory since one of the major problems of sting theory is to cure the undesirable singularity in general relativity.

In the earlier works on the solutions in the EBI system, the dilaton and the axion field are not taken into account to the best of our knowledge. We should not, however, ignore them from the point of view of string theory (especially of the latter context) since it is well known that the GM-GHS solution has many different aspects from the Reissner-Nortström (RN) black hole in the Einstein-Maxwell system. Our main purpose in this letter is to survey the particle-like and black hole solutions in the Einstein-Born-Infeld-dilaton(-axion) (EBID)

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system, which we call dilatonic EBion (DEBion) and dilatonic EBion black hole (DEBion black hole), respectively, and to clarify the effect of the dilaton field.

Model and Basic Equations: We start with the following action [15];

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{(\nabla\phi)^2}{\kappa^2} - \frac{1}{24\kappa^2} e^{-4\gamma\phi} H^2 + L_{BI} \right], \quad (1)$$

where $\kappa^2 := 8\pi$ and γ is the coupling constant of the dilaton field ϕ . The three rank antisymmetric tensor field $H^2 = H_{\mu\nu\rho} H^{\mu\nu\rho}$ is expressed as, $H = dB + \frac{1}{4} A \wedge F$. L_{BI} is the BI part of the Lagrangian which is written as

$$L_{BI} = \frac{be^{2\gamma\phi}}{4\pi} \left\{ 1 - \sqrt{1 + \frac{e^{-4\gamma\phi}}{2b} P - \frac{e^{-8\gamma\phi}}{16b^2} Q^2} \right\}, \quad (2)$$

where $P := F_{\mu\nu} F^{\mu\nu}$ and $Q := F_{\mu\nu} \tilde{F}^{\mu\nu}$. A tilde denotes the Hodge dual. Since we examine the electric and the magnetic monopole cases separately, the term Q vanishes and eventually the axion field becomes trivial by putting the spherically symmetric ansatz. The dyon case including the axion field are summarized in elsewhere [17]. The BI parameter b has physical interpretation of a critical field strength. In the string theoretical context, b is related to the inverse string tension α' by $b^{-1} = (2\pi\alpha')^2$. Notice that the action (1) reduces to the EMD system in the limit $b \rightarrow \infty$ and to the EBI system with the massless field for $\gamma = 0$. Here, we concentrate on the case $\gamma = 1$ which is predicted from superstring theory and $\gamma = 0$ for comparison.

We consider the metric of static and spherically symmetric,

$$ds^2 = -f(r)e^{-2\delta(r)} dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega^2, \quad (3)$$

where $f(r) = 1 - 2m(r)/r$. The gauge potential has the following form:

$$A = -\frac{w_1(r)}{r} dt - w_2(r) \cos\theta d\varphi. \quad (4)$$

Since we do not consider the dyon solution, we drop the term w_2 (or w_1) in the electrically (or magnetically) charged case. From the BI equation, we obtain that $w_2 \equiv Q_m$ is constant and

$$\left(\frac{w_1}{r} \right)' = -Q_e e^{2\phi-\delta} \left(r^4 + \frac{Q_e^2}{b} \right)^{-\frac{1}{2}}, \quad (5)$$

where $Q_e := w_1(\infty)$ and a prime denotes a derivative with respect to r . From this equation, the electric field $E_r = -(w_1/r)'$ does not diverge but takes the finite value at the origin. The maximum value is $E_r = \sqrt{b}$ in the EBI system. As we will see, the electric field vanishes at the origin in the EBID system by the nontrivial behavior of ϕ . The potential w_1 is formally expressed as

$$w_1 = -r \int_0^r \frac{Q_e e^{2\phi-\delta}}{r^2} \left(r^4 + \frac{Q_e^2}{b} \right)^{-\frac{1}{2}} dr, \quad (6)$$

where we put $rw_1(0) = 0$ without loss of generality.

By the above ansätze the basic equations with $\gamma = 1$ are written as follows.

$$m' = -U + \frac{r^2}{2} f(\phi')^2, \quad (7)$$

$$\delta' = -r(\phi')^2, \quad (8)$$

$$\phi'' = -\frac{2}{r}\phi' - \frac{2}{f} \left[\left(\frac{m}{r} + U \right) \frac{\phi'}{r} - X \right], \quad (9)$$

where

$$U := e^{2\phi} \left(br^2 - \sqrt{b^2 r^4 + bQ_e^2 + bQ_m^2 e^{-4\phi}} \right), \quad (10)$$

$$X := be^{2\phi} \left(\sqrt{\frac{br^4 + Q_e^2}{br^4 + Q_m^2 e^{-4\phi}}} - 1 \right). \quad (11)$$

We drop the terms which do not vanish in the dyon case but do in the monopole case. Note that by introducing the dimensionless variables $\bar{r} := \sqrt{b}r$, $\bar{m} := \sqrt{b}m$, $\alpha_e := \sqrt{b}Q_e$ and $\alpha_m := \sqrt{b}Q_m$, we find that the parameters of the equation system are α_e and α_m .

As was already pointed out by Gibbons and Rasheed [18], this system loses electric-magnetic duality, although the modification to satisfy the $SL(2, \mathbb{R})$ duality is also considered [19]. Hence, when we obtained the black hole solution with electric charge in the above action, we can not transform to the solution with magnetic charge by duality. We are interested in the difference between the black holes with the electric charge and those with the magnetic charge.

The boundary conditions at spatial infinity to satisfy the asymptotic flatness are

$$m(\infty) \equiv M = \text{Const.}, \quad \delta(\infty) = 0, \quad \phi(\infty) = 0. \quad (12)$$

We also assume the existence of a regular event horizon at $r = r_h$ for the DEBion black hole. So we have

$$m_h = \frac{r_h}{2}, \quad \delta_h < \infty, \quad \phi_h < \infty, \quad \phi'_h = \frac{2r_h X_h}{(1 + 2U_h)}. \quad (13)$$

The variables with subscript h means that they are evaluated at the horizon. Under these conditions, we obtained the black hole solution numerically.

Electrically charged solution: First, we investigate the electrically charged case. Before proceeding to the DEBion black hole, we briefly review the solutions in the EBI system ($\gamma = 0$). In the $b \rightarrow \infty$ limit, the EBI system reduces to the EM system and the RN black hole is the solution of the Eqs. (7)-(9). For the finite b , we obtain EBion and its black hole solutions analytically [7,8]. The metric functions are written as $\delta \equiv 0$ and

$$m(r) = m_0 + \frac{bQ_e^2}{3} \left[\frac{r}{r^2 + \sqrt{r^4 + bQ_e^2}} + \frac{1}{\sqrt{bQ_e}} F\left(\frac{1}{\sqrt{2}}, \arccos \frac{\sqrt{bQ_e} - r^2}{\sqrt{bQ_e} + r^2}\right) \right], \quad (14)$$

where $F(k, \varphi)$ is the elliptic function of the first kind. The constant m_0 is the mass of the central object. $m_0 = 0$ corresponds to the EBion solution. As is seen from the metric function, the number of the horizon depends on the parameter α_e and m_0 (or M). We plot the M - r_h relation in Fig. 1. The solution branches are divided qualitatively by $\alpha_e = \alpha^* := 1/2$. For $\alpha_e > \alpha^*$, there is a special value M_0 of which the analytic form is seen in Ref. [8]. For $M < M_0$ the black hole and inner horizon exist as the RN black hole while only the black hole horizon exists for $M \geq M_0$. The minimum mass solution in each branch corresponds to the extreme solution. On the other hand, all the black hole solutions have only one horizon and the global structure is the same as the Schwarzschild black hole for $\alpha_e < \alpha^*$. In this case we can take the $r_h \rightarrow 0$ limit. These solutions correspond to the EBion solution with no horizon. Note however that, since $m'(0) = \alpha_e$, they have the conical singularity at the origin, which is the characteristic feature of the self-gravitating Bions. For $\alpha_e = \alpha^*$, the extreme solution is realized in the $r_h \rightarrow 0$ limit. Although Demianski called it an electromagnetic geon [7] which is regular everywhere, it has a conical singularity like the other EBions.

Next we turn to the $\gamma = 1$ case. In the $b \rightarrow \infty$ limit, i.e., in the EMD system, the GM-GHS solution exists. The GM-GHS solution has three global charges, i.e., mass, the electric charge and the dilaton charge. The last one depends on the former two, hence the dilaton charge is classified into the secondary charge [20]. Although the GM-GHS solution has the electric charge as the RN solution, it does not have an inner horizon but has spacelike singularity inside the event horizon due to the effect of the dilaton field. In the extreme limit, the event horizon coincides with the central singularity and a naked singularity appears at the origin [14]. There is no particle-like solutions in this system.

For the finite value of b , we can not find the analytic solution and use the numerical analysis. Since there is no non-trivial dilaton configuration in the $Q_e = 0$ case, the dilaton hair is again the secondary hair in this system. We plot the M - r_h relation of the DEBion black hole in Fig. 1. We can find that all branches reach to $r_h = 0$ in contrast to the EBion case. This is explained as follows. The extremal solution has a degenerate horizon and $2m' = 1$ is realized on the horizon. Hence, by Eq. (7)

$$\alpha_e^{ext} = \sqrt{\frac{e^{-4\phi_h}}{4} + br_h^2 e^{-2\phi_h}}. \quad (15)$$

Since $\phi_h = 0$ in the EBI system, the extreme solutions exist for $\alpha_e \geq \alpha^*$. We will examine the behavior of the

dilation field around the horizon. On the horizon, the equation of the dilaton field becomes $f'_h \phi'_h = 2X_h$. Since $f'_h > 0$ and X is positive except for the origin in the electrically charged case, the dilaton field increases around the horizon. Next, we assume that there is an extremum point of the dilaton field. At this point, the equation of the dilaton field becomes $f\phi'' = 2X$, which implies that the extremum point must be a local minimum. From this behavior, we find that if $\phi_h \geq 0$ the dilaton field can not approach its asymptotic value $\phi(\infty) = 0$. As a result, $\phi_h < 0$. This means that the value α_e for which the extreme solution exists must be larger than that in the EBI system. Furthermore, ϕ_h diverges to minus infinity in the $r_h \rightarrow 0$ limit as we will see below. As a result $\alpha_e^{ext} \rightarrow \infty$ and no extreme solution exists for finite α_e .

As for the ‘particle-like’ solution, which is not a relevant one we show below, we have to analyze carefully. By performing Taylor expansion for the basic equations around the origin to see the behavior of the dilaton fields, we find that the lowest order equation is inconsistent. This implies that the dilaton field diverges at the origin. Then we employ the new function $\psi := e^{2\phi}$ and expand the field variables as

$$\psi = \sum_{\alpha, \beta} \psi_{(\alpha, \beta)} r^\alpha (\ln r)^\beta, \quad m = \sum_{\gamma, \delta} m_{(\gamma, \delta)} r^\gamma (\ln r)^\delta. \quad (16)$$

Substituting them into the basic equations and evaluating the lowest order equations, we find

$$\phi \sim -\frac{1}{2} \ln(-4\sqrt{bQ_e^2} \ln r), \quad m \sim -\frac{r}{4 \ln r}. \quad (17)$$

Note that the singular behavior of the mass function becomes mild by a factor $1/\ln r$ and that the lapse function δ remains finite in spite of the divergence of the dilaton field.

The diverging behavior of the dilaton field is important when we consider the solutions in the original string frame. Since the conformal factor is $e^{-2\phi}$, the conformal transformation becomes singular at the origin, and there is a possibility that such transformation gives pathological behavior. Actually, performing conformal transformation back to the string frame, we find that the metric is degenerate at the origin, and that the strong curvature singularity occurs. In this sense, the ‘particle-like’ solution we found is not a relevant one.

By integrating from the event horizon back to the origin with suitable boundary conditions, we can examine the internal structure of the black hole solutions. The dilaton field monotonically decreases and diverges as $\approx \log r$. The electric field vanishes approaching to the origin by the divergence of the dilaton field. The mass function m also diverges as $\approx r^{-x}$, ($0 < x < 1$). Hence, the function f does not have zero except at the event horizon. As a result, the global structure is Schwarzschild type.

The temperature of the DEBion black holes is expressed by

$$T = \frac{e^{-\delta_h}}{4\pi r_h} (1 + 2U_h). \quad (18)$$

We show them in Fig. 2. The DEBion black hole has always a higher temperature than the GM-GHS solution ($T = 1/8\pi M$) since the field strength of the gauge field becomes mild by the BI field. The specific heat of the DEBion black hole is always negative while the discontinuous change of the sign of the heat capacity occurs once or twice for the EBion black holes depending on α_e . Since the EBion black hole has an extreme limit for $\alpha_e > \alpha^*$, the temperature becomes zero in this limit. On the other hand, since there is no extremal solution for DEBion black holes, their temperature does not go to zero but diverges in the $r_h \rightarrow 0$ limit. Hence the evolution by the Hawking evaporation does not stop until the singular solution with $r_h \rightarrow 0$ when the surrounding matter field does not exist.

Magnetically charged solution: Tuning now to the magnetically charged case. EBion and GM-GHS black holes with magnetic charge can be obtained from the following duality as $F \rightarrow \tilde{F}$, and $F \rightarrow e^{-2\phi} \tilde{F}$, $\phi \rightarrow -\phi$, respectively. So the relations M - r_h and M - $1/T$ never change. As is noted above however, our DEBion black hole has no such duality. So we can expect that the magnetically charged solutions have different properties from electrically charged one.

From Eq. (7),

$$\alpha_m^{ext} = \sqrt{\frac{1}{4} + br_h^2 e^{2\phi_h}} \quad (19)$$

for the extreme solutions. Thus, there is no extreme solution for $\alpha_m < 1/2$. For $\alpha_m \geq 1/2$, we must survey ϕ_h . For $\alpha_m = 1/2$, because $r_h^2 e^{2\phi_h} \rightarrow 0$ as $r_h \rightarrow 0$, extreme solution is realized in the $r_h \rightarrow 0$ limit, i.e., $\alpha_m = \alpha_m^{ext}$. For $\alpha_m > 1/2$, $r_h^2 e^{2\phi_h} \rightarrow \text{Const.}$ as $r_h \rightarrow 0$. We cannot tell whether Eq. (19) is fulfilled for a certain horizon radius since ϕ_h is obtained iteratively only by numerical method. Our calculation always shows $\alpha_m < \sqrt{1/4 + br_h^2 e^{2\phi_h}}$ except for $\alpha_m = \infty$.

Although the M - r_h relation seems similar to that of the GM-GHS solution regardless of α_m (Fig. 3), the M - $1/T$ relation is different depending on α_m (Fig. 4). This is due to the qualitative difference of the behavior of the dilaton field in the $r_h \rightarrow 0$ limit. For $\alpha_m \geq \alpha^*$, the dilaton field on the horizon diverges as $\phi_h \approx \ln r_h$ and $e^{-\delta_h} \rightarrow r_h$, and hence the temperature remains finite as in the GM-GHS case. On the other hand, ϕ_h and δ_h are finite in the $r_h \rightarrow 0$ limit for $\alpha_m < \alpha^*$. As a result, the temperature diverges. Note that the corresponding solution is the particle-like solution. The remarkable feature is that the dilaton field is finite everywhere. Hence it is also the relevant particle-like solution in the original string frame. In the magnetically charged case, DEBion solution does exist for $\alpha_m < \alpha^*$.

The behaviors of the field functions inside the event horizon are similar to those in the electrically charged

case qualitatively except for the sign of the dilaton field. The magnetic field diverges as $B_r \approx r^{-x}$, ($1 < x < 2$).

Discussion: We investigate the static spherically symmetric solutions in the EBID system. In the electrically charged case, there is neither extreme solution nor particle-like solution both of which exist in the EBI system. The temperature diverges monotonically in the $r_h \rightarrow 0$ limit. On the other hand, the extreme solution exists when $\sqrt{b}Q_m = 1/2$ and particle-like solutions do when $\sqrt{b}Q_m < 1/2$ for the magnetically charged case. For $\sqrt{b}Q_m > 1/2$, the solution in the $r_h \rightarrow 0$ limit corresponds to naked singularity. The behavior of the temperatures varies depending on $\sqrt{b}Q_m$. We do not find the inner horizon in any charged case, and the global structure is the Schwarzschild type. This is due to the effect of the dilaton field.

Here, we discuss some outstanding issues. One of the most important issues is the stability of the new solutions. Here we briefly discuss this by using catastrophe theory [21,22], which is a useful tool to investigate relative stability. Adopting M , Q_e , (or Q_m), b , F_h^2 and r_h as catastrophe variables, we find that the equilibrium space becomes single valued space with respect to the control parameters and includes the stable GM-GHS solution. This implies that DEBion black hole is also stable at least against spherical perturbations.

We also discuss the possibility of relating inverse string tension α' and the gravitational constant G . Following the lines in Ref. [18], the supersymmetric spin $1/2$, 0 particle is described by BI action if we choose

$$\frac{1}{b} = \frac{2}{3} \left(\frac{e}{m} \right)^4, \quad (20)$$

where e and m is the U(1) charge and the mass of the particle, respectively. Identifying the particle with extreme black hole solution and approximating it to the RN solution, we obtain $2\pi\alpha' \sim \sqrt{2/3}G$. If we apply this discussion to the system including BI action, it may be suitable using e/m ratio of the EBion black hole and DEBion black holes with α_e (or α_m) = α^* , in which cases, extreme solution is realized in the zero horizon limit. Then, we find $2\pi\alpha' \sim 1.07G$ for EBion black hole, and $2\pi\alpha' \sim 1.73G$ for DEBion black hole. So the discussion in [18] is not much affected. More detailed properties and the dyon case including the axion are shown in our next paper [17].

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- [1] M. Born and L. Infeld, Proc. Roy. Soc. A **144**, 425, (1934).
- [2] A. Tseytlin, Nucl. Phys. B **501**, 41 (1997); D. Brecher, Phys. Lett. B **442**, (1998); S. Gonorazky, F. Schaposnik and G. Silva, Phys. Lett. B **449**, 187 (1999); J. Park, Phys. Lett. B **458**, 471 (1999); For a review, see hep-th/9908105 and hep-th/9909154.
- [3] D. Gal'tsov and R. Kerner, hep-th/9910171.
- [4] Black hole solutions in the SU(2) EBI system are also reported recently [5].
- [5] M. Wirschins, A. Sood and J. Kunz, hep-th/0004130; V. V. Dyadichev and D. V. Gal'tsov, hep-th/0005099.
- [6] In this word we mean the solution which has conical singularity at the origin and has finite energy.
- [7] M. Demianski, Found. Phys. **16**, 187 (1986); D. Wiltshire, Phys. Rev. D **38**, 2445 (1988).
- [8] H. d' Oliveira, Class. Quant. Grav. **11**, 1469 (1994).
- [9] H. Salazar, A. García and J. Plebański, J. Math. Phys. **28**, 2171 (1987).
- [10] E. Ayón-Beato and A. García, Phys. Rev. Lett. **80**, 5056 (1998); *ibid.*, Gen. Rel. Grav. **31**, 629 (1999); *ibid.*, Phys. Lett. B **464**, 25 (1999).
- [11] J. Bardeen, in Proceedings of GR5, Tiflis, U. S. S. R., 1968; A. Borde, Phys. Rev. D **55**, 7615 (1997).
- [12] J. Polchinski, hep-th/9611050; M. Aganagic, J. Park, C. Popescu and J. H. Schwarz, Nucl. Phys. B **496**, 191 (1997); A. Tseytlin, Nucl. Phys. B **469**, 51 (1996).
- [13] E. Fradkin and A. Tseytlin, Phys. Lett. B **163**, 123 (1985); A. Tseytlin, Nucl. Phys. B **276**, 391 (1986).
- [14] G. W. Gibbons and K. Maeda, Nucl. Phys. B **298**, 741 (1988); D. Garfinkle, G. T. Horowitz and A. Strominger, Phys. Rev. D **43**, 3140 (1991).
- [15] In this paper, we study the tree-level action. When we are interested in the higher-loop action, the coupling constant γ takes different value by assuming anomaly cancellation (See. Ref. [16]).
- [16] E. Bergshoeff, E. Sezgin, C. Pope and P. Townsend, Phys. Lett. B **188**, 70 (1987); R. Metsaev, M. Rahmanov and A. Tseytlin, Phys. Lett. B **193**, 207 (1987); C. Callan, C. Lovelace, C. Nappi and S. Yost, Nucl. Phys. B **308**, 221 (1988); O. Andreev and A. Tseytlin, Nucl. Phys. B **311**, 205 (1988).
- [17] T. Tamaki and T. Torii, in preparation.
- [18] G. W. Gibbons and D. A. Rasheed, Nucl. Phys. B **454**, 185 (1995).
- [19] D. A. Rasheed, Phys. Lett. B **365**, 46 (1996).
- [20] P. Bizon, Acta. Phys. Pol. B **25**, 877 (1994).
- [21] R. Thom, *Structural Stability and Morphogenesis* (Benjamin, New York, 1975).
- [22] K. Maeda, T. Tachizawa, T. Torii and T. Maki, Phys. Rev. Lett. **72**, 450 (1994); T. Torii, K. Maeda and T. Tachizawa, Phys. Rev. D **51**, 1510 (1995); T. Tachizawa, K. Maeda and T. Torii, Phys. Rev. D **51**, 4054 (1995).

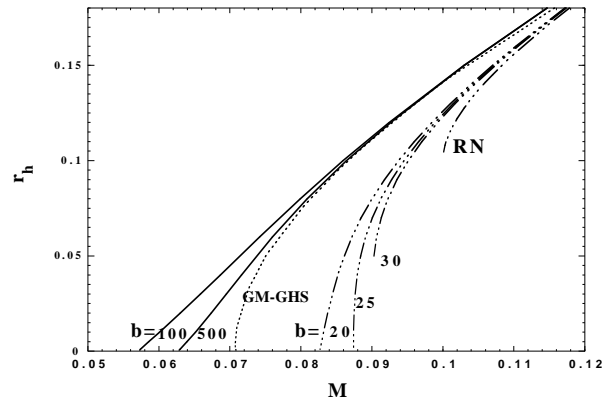


FIG. 1. M - r_h diagram of the electrically charged DEBion black holes (solid lines) with electric charge $Q_e = 0.1$ and $b = 100$ and 500 . GM-GHS and EBion black holes are also plotted by dotted line and the dot-dashed lines, respectively.

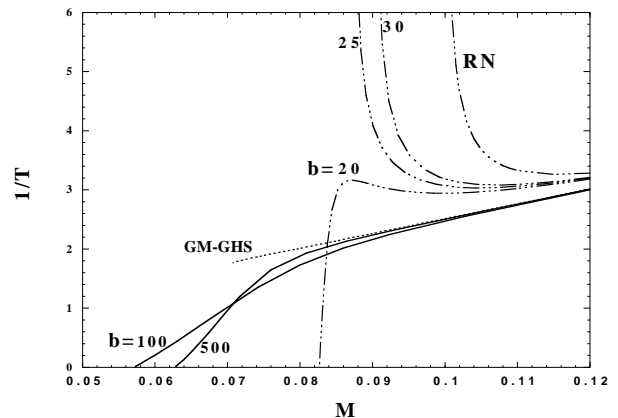


FIG. 2. M - $1/T$ diagram of the electrically charged DEBion black holes (solid lines) with the same parameters in Fig. 1.

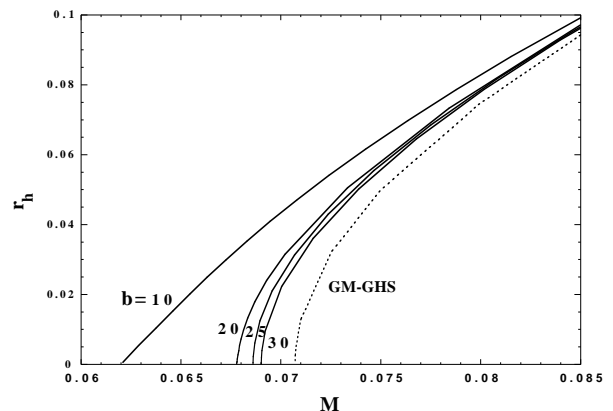


FIG. 3. M - r_h diagram of the magnetically charged DEBion black holes (solid lines) with magnetic charge $Q_m = 0.1$ and $b = 10, 20, 25$ and 30 . GM-GHS black hole is also plotted by dotted line.

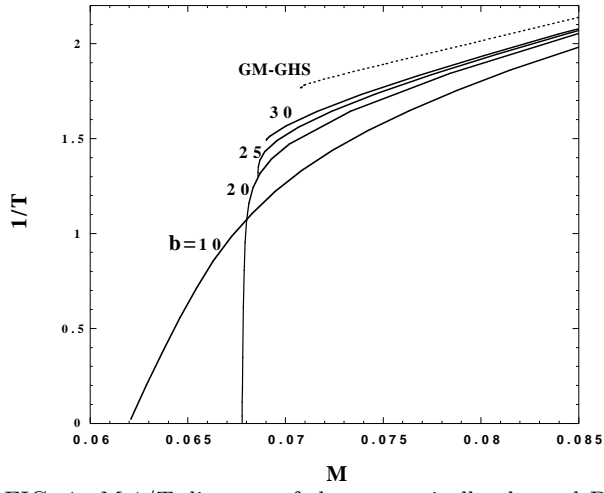


FIG. 4. M - $1/T$ diagram of the magnetically charged DE-BIon black holes (solid lines) with the same parameters in Fig. 3.